Neutralinos in E_6 inspired supersymmetric $U(1)'$ models

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Abstract. The neutralino sector in E_6 inspired supersymmetric models with extra neutral gauge bosons and singlet Higgs fields contains additional gaugino and singlino states compared to the MSSM. We discuss the neutralino mixing in rank-5 and rank-6 models and analyze the supersymmetric parameter space where the light neutralinos have mainly singlino or MSSM character. The neutralino character, resonance effects of the new gauge bosons and, assuming mSUGRA-type RGEs, different selectron masses lead to significant differences between the MSSM and the extended models in neutralino production at an e^+e^- linear collider. Beam polarization may improve the signatures to distinguish between the models. In an appendix, we present the mass terms of the gauge bosons, charginos and sfermions which show a significant different mass spectrum than in the MSSM and give all relevant neutralino couplings.

1 Introduction

Supersymmetry is considered to be one of the most fascinating concepts in particle physics. It may provide solutions to the hierarchy and fine tuning problems of the Standard Model (SM) and allows unification of the coupling constants at the scale E_{GUT} of a Grand Unified Theory (GUT). In the Minimal Supersymmetric Standard Model (MSSM) [1] one has to explain the weak scale value of the μ parameter in the superpotential [2]. Supersymmetric models with additional singlet Higgs fields evade the μ problem by replacing μ with a product of a dimensionless coupling and a vacuum expectation value of a singlet Higgs field. The simplest extension of the MSSM by one additional singlet Higgs field is the Next-to-minimal Supersymmetric Standard Model (NMSSM) [3].

Models with additional $U(1)$ factors in the gauge group containing new neutral gauge bosons are a further extension of the MSSM [4–6]. These models provide a solution to the domain-wall problem of the NMSSM [7], because the discrete $\mathbb{Z}_3$ symmetry is embedded in the new $U(1)$ factors [4,6]. One or two additional $U(1)$ factors can be motivated by the breaking of an E_6 group which is a good candidate of an (effective) GUT group [8]. Since experimental results lead to strict lower mass bounds for the new gauge bosons, further studies are needed in order to understand the hierarchy between the gauge boson masses [9]. Also the SUSY breaking scale is typically of the order of the new gauge bosons. However, gaugino and slepton masses of some 100 GeV at the electroweak scale are not excluded [10,11]. The extended neutralino sector in these E_6 inspired $U(1)'$ models will be discussed in detail in this work focusing especially on the differences to the MSSM.

The production of neutralinos at an electron-positron linear collider with polarized beams is considered as an excellent process to discriminate between the supersymmetric models. The experimental signatures certainly depend on the neutralino decay channels which are discussed for singlino-like lightest supersymmetric particles (LSP) in [12]. Especially one expects the existence of displaced decay vertices for large singlet vacuum expectation values [13].

We discuss the neutralino phenomenology in two types of scenarios where the mass of the new gauge bosons is above the reach of the first generation of linear colliders with 500 to 800 GeV center-of-mass energy. Within the framework of constrained E_6 models with GUT relations between the soft supersymmetry breaking masses of order 100 GeV the lighter four neutralinos have MSSM-like character. Assuming mSUGRA-type RGE relations for the sfermion masses, the masses of the left and right selectrons in the constrained E_6 models considerably differ from the ones in the constrained MSSM. Especially if the selectrons cannot be directly produced at a linear collider, neutralino production may offer valuable information about the underlying supersymmetric model. The selectron RGE relations in E_6 models can be tested in scenarios with gaugino-like light neutralinos while particularly the polarization asymmetries of the production cross sections of higgsino dominated neutralinos show new gauge boson resonance effects at energies well below their masses.

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Table 1. Rank of the gauge group, new factors in the gauge group, number of the singlet Higgs fields obtaining a vev, resulting number of the neutralinos, soft breaking gaugino mass parameters and singlet vevs in the considered models R5$_1$ (rank-5 model with one singlet), R5$_2$ (rank-5 model with two singlets) and R6 (rank-6 model)

Model	rank	new gauge factors	singlet Higgs	neutralinos	soft breaking parameters	singlet vevs
R5$_1$	5	U(1)$'$	1	6	M_2, M_1, M'	v_3
R5$_2$	5	U(1)$'$	2	7	M_2, M_1, M'	v_3, v_4
R6	6	U(1)$' \times$ U(1)$''$	2	8	M_2, M_1, M', M''	v_3, v_4

In a second type of scenarios we relax the GUT and RGE relations and obtain singlino-like light neutralinos for large values of the soft breaking U(1)$'$ gaugino mass parameter. Then neutralino production provides a favorable way to determine the neutralino character and the parameters of the underlying supersymmetric model.

This paper is organized as follows: In Sect. 2 we analyze the neutralino sector of the E_6 inspired models with new U(1) factors and discuss the mass spectra of the neutralinos in these models which contain up to four exotic singlino and new gaugino states. In Sect. 3 the production of neutralinos in electron-positron collisions is analyzed in representative scenarios where the lightest MSSM-like neutralino has gaugino or higgsino character. We work out differences of the cross sections and polarization asymmetries between the MSSM and the extended models for MSSM-like light neutralinos and discuss as well the production of exotic singlino-like neutralinos. In the appendix we give a brief overview of the E_6 models focusing mainly on the breaking of the E_6 group resulting in new U(1) factors and on the particle content. We discuss the mass terms of the gauge bosons, charginos and sfermions, which have a significantly different mass spectrum than in the MSSM and the NMSSM because of additional D-terms of the new U(1) factors. The relevant couplings for production and decay of the neutralinos in the E_6 models can also be found in the appendix.

2 Neutralino mass spectra

The additional gauge bosons and singlet Higgs fields in E_6 inspired models lead to an extended neutralino sector which may be crucial in order to distinguish between these models and the MSSM or NMSSM in future high energy collider experiments. The breaking of a GUT group E_6 can lead to low energy gauge groups with one (rank 5) or two (rank 6) additional U(1) factors in comparison to the SM (App. A). The particle spectrum of the E_6 models contains two neutral SM singlet fields which can be interpreted as singlet Higgs fields (App. B). The vacuum expectation values (vevs) of these singlets break the new U(1) factors and create masses for the corresponding new gauge bosons large enough to respect the experimental bounds (App. C). We assume the absence of gauge kinetic mixing between the U(1) factors in the discussed models.

In the *rank-5 model with one singlet* (R5$_1$) only one of the singlet fields obtains a vev, whereas in the *rank-5 model with two singlets* (R5$_2$) both singlets vevs are present. In the *rank-6 model* (R6) both vevs are necessary to create the masses for both new gauge bosons. The neutralino masses and mixings in these models summarized in Table 1 depend on the soft symmetry breaking gaugino mass parameters, the ratio $\tan\beta = v_2/v_1$ of the doublet Higgs vevs, the singlet Higgs vevs and the trilinear coupling λ of the superpotential term $W_\lambda = \lambda H_1 H_2 N_1$ which replaces the μ term of the MSSM. W_λ is the only superpotential term relevant for the mass terms of the neutralinos. Terms $\sim H_1 H_2 N_2$ or $\sim N_1^\dagger N_2$, where H_1, H_2, N_1, N_2 are the doublet and singlet Higgs fields defined in App. B, are forbidden by the E_6 gauge symmetry [8, 14–16]. Thus the product of the dimensionless coupling λ with the singlet vev v_3 becomes the effective μ parameter in the E_6 models.

The neutralino masses and mixings can be derived from the most general neutralino mass term in the Lagrangian of the rank-6 model

$$
\begin{aligned}
\mathcal{L}_{m_{\chi^0}} = {} & \frac{1}{\sqrt{2}} i g_2 \lambda^3 (v_1 \psi_{H_1}^1 - v_2 \psi_{H_2}^2) \\
& - \frac{1}{\sqrt{2}} i g_1 \lambda_1 (v_1 \psi_{H_1}^1 - v_2 \psi_{H_2}^2) + \frac{1}{\sqrt{2}} i g' \lambda' \Big(Y_1' v_1 \psi_{H_1}^1 \\
& + Y_2' v_2 \psi_{H_2}^2 + Y_3' v_3 \psi_{N_1} + Y_4' v_4 \psi_{N_2} \Big) + \frac{1}{\sqrt{2}} i g'' \lambda'' \\
& \times \Big(Y_1'' v_1 \psi_{H_1}^1 + Y_2'' v_2 \psi_{H_2}^2 + Y_3'' v_3 \psi_{N_1} + Y_4'' v_4 \psi_{N_2} \Big) \\
& + \frac{1}{2} M_2 \lambda^3 \lambda^3 + \frac{1}{2} M_1 \lambda_1 \lambda_1 + \frac{1}{2} M' \lambda' \lambda' + \frac{1}{2} M'' \lambda'' \lambda'' \\
& - \lambda v_3 \psi_{H_1}^1 \psi_{H_2}^2 - \lambda v_1 \psi_{H_2}^2 \psi_{N_1} - \lambda v_2 \psi_{H_1}^1 \psi_{N_1} \\
& + \text{h.c.} \ ,
\end{aligned}
\tag{1}
$$

with the two component Weyl spinors λ^3, λ_1, λ' and λ'' of the neutral SU(2)$_L$-, U(1)$_Y$-, U(1)$'$ and U(1)$''$ gauginos and $\psi_{H_1}^1$, $\psi_{H_2}^2$, ψ_{N_1} and ψ_{N_2} of the doublet and singlet higgsinos (singlinos), respectively. The Y_i' (Y_i'') are the U(1)$'$ (U(1)$''$) quantum numbers of the doublet and singlet Higgs fields. In the following we will assume U(1)$' \equiv$ U(1)$_\eta$ in the rank-5 models, so in R5$_1$ and R5$_2$

$$
Y_{1,2}' = Y_\eta(H_{1,2}), \qquad Y_{3,4}' = Y_\eta(N_{1,2}), \tag{2}
$$

as given in Table 6. In the rank-6 model (R6) it is U(1)$' \equiv$ U(1)$_\psi$ and U(1)$'' \equiv$ U(1)$_\chi$, so with Table 6

$$
Y_{1,2}^{'('')} = Y_{\psi(\chi)}(H_{1,2}), \qquad Y_{3,4}^{'('')} = Y_{\psi(\chi)}(N_{1,2}). \tag{3}
$$

$$Y = \begin{pmatrix} Y_{11} & Y_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ Y_{12} & Y_{22} & m_{Z^{\mathrm{SM}}} & 0 & 0 & 0 & 0 & 0 \\ 0 & m_{Z^{\mathrm{SM}}} & -\lambda v_3 \sin 2\beta & \lambda v_3 \cos 2\beta & Y_{35} & 0 & 0 & Y_{38} \\ 0 & 0 & \lambda v_3 \cos 2\beta & \lambda v_3 \sin 2\beta & Y_{45} & \lambda v & 0 & Y_{48} \\ 0 & 0 & Y_{35} & Y_{45} & M' & Y_3' \dfrac{g' v_3}{\sqrt{2}} & Y_4' \dfrac{g' v_4}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & \lambda v & Y_3' \dfrac{g' v_3}{\sqrt{2}} & 0 & 0 & Y_3'' \dfrac{g'' v_3}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & Y_4' \dfrac{g' v_4}{\sqrt{2}} & 0 & 0 & Y_4'' \dfrac{g'' v_4}{\sqrt{2}} \\ 0 & 0 & Y_{38} & Y_{48} & 0 & Y_3'' \dfrac{g'' v_3}{\sqrt{2}} & Y_4'' \dfrac{g'' v_4}{\sqrt{2}} & M'' \end{pmatrix} \tag{6}$$

With the assumption that all U(1) factors are created at the same energy scale, e.g. at the scale where an underlying E_6 group is broken, and obey the same renormalization group equations the couplings g_1, g' and g'' should have the same value at the electroweak scale [8,15,17]. In the remainder of this paper we assume $g' = g'' = g_1$ in all numerical discussions.

Then in the basis

$$(\psi^0)^T = (-i\lambda_\gamma, -i\lambda_Z, \psi_H^a, \psi_H^b, -i\lambda', \psi_{N_1} [, \psi_{N_2} [, -i\lambda'']]) \tag{4}$$

with

$$\lambda_\gamma = \lambda^3 \sin\theta_W + \lambda_1 \cos\theta_W ,$$
$$\lambda_Z = \lambda^3 \cos\theta_W - \lambda_1 \sin\theta_W ,$$
$$\psi_H^a = \psi_{H_1}^1 \cos\beta - \psi_{H_2}^2 \sin\beta , \tag{5}$$
$$\psi_H^b = \psi_{H_1}^1 \sin\beta + \psi_{H_2}^2 \cos\beta ,$$

the neutralino mass term in the rank-6 model becomes $\mathcal{L}_{m_{\chi^0}} = -\frac{1}{2}(\psi^0)^T Y \psi^0 +$ h.c. with the 8×8 neutralino mass matrix (see (6) on top of the page) where the matrix elements are given by

$$Y_{11} = M_2 \sin^2\theta_W + M_1 \cos^2\theta_W ,$$
$$Y_{12} = (M_2 - M_1) \sin\theta_W \cos\theta_W ,$$
$$Y_{22} = M_2 \cos^2\theta_W + M_1 \sin^2\theta_W ,$$
$$Y_{35} = \frac{g'v}{\sqrt{2}} \left(Y_1' \cos^2\beta - Y_2' \sin^2\beta \right) ,$$
$$Y_{38} = \frac{g''v}{\sqrt{2}} \left(Y_1'' \cos^2\beta - Y_2'' \sin^2\beta \right) ,$$
$$Y_{45} = \frac{g'v}{2\sqrt{2}} (Y_1' + Y_2') \sin 2\beta ,$$
$$Y_{48} = \frac{g''v}{2\sqrt{2}} (Y_1'' + Y_2'') \sin 2\beta ,$$
$$v \equiv \sqrt{v_1^2 + v_2^2} = \sqrt{2}\, \frac{m_W}{g_2} . \tag{7}$$

Assuming CP conservation, all parameters are real. The physical masses of the neutralinos can be derived by diagonalization with a real orthogonal 8×8 matrix N [1, 18]

$$\eta_{\tilde\chi_i^0} m_{\tilde\chi_i^0} \delta_{ik} = N_{im} N_{kn} Y_{mn} , \tag{8}$$

with the physical masses $m_{\tilde\chi_i^0}$ of the neutralinos and the sign factors $\eta_{\tilde\chi_i^0}$ of the respective eigenvalues. The 6×6 and 7×7 neutralino mixing matrices of the models $R5_1$ and $R5_2$, respectively, are obtained as the upper left 6×6 and 7×7 submatrices of Y as shown in (6) [4,15,19–25].

The upper left 4×4 submatrix contains the mixing matrix of the MSSM if $-\lambda v_3$ is replaced by the μ parameter. The lower right submatrix (2×2, 3×3 and 4×4, respectively) represents the new exotic components of the neutralinos in the E_6 models. The entries in the nondiagonal submatrices are of the order of the doublet vacuum expectation values and therefore of $m_{Z^{\mathrm{SM}}}$. On the other hand in the "exotic" submatrix most entries are of the order of the singlet vacuum expectation values and therefore of $m_{Z'}$ and $m_{Z''}$. This results in an approximate decoupling of the exotic neutralinos from the MSSM-like ones as shown in Figs. 1a and e for the models $R5_1$ and R6, respectively, with GUT relation $M' = M'' = M_1 = M_2\, 5/3 \tan^2\theta_W$ for the gaugino mass parameters. With $M_2, \lambda v_3 = \mathcal{O}(100\ \mathrm{GeV})$, the four lighter neutralinos are mainly MSSM-like while the heavy neutralinos have exotic character with masses of the order of $m_{Z'}$ and $m_{Z''}$. In model $R5_1$ (Fig. 1a) the masses of the two exotic neutralinos are approximately

$$m_{\tilde\chi_{5,6}^0} \approx Y_3' \frac{g' v_3}{\sqrt{2}} \pm \frac{1}{2} M' = Y_3' \frac{g' v_3}{\sqrt{2}} \pm M_2 \frac{5}{6} \tan^2\theta_W . \tag{9}$$

Equation (9) is valid for all small $M' \ll v_3$, including $M' < M_1$. Hence both corresponding exotic neutralinos always have masses of order $m_{Z'}$ and are mixtures of the singlino and Z' gaugino eigenstates. For $M_1, M_2, \lambda v_3 = \mathcal{O}(v_3, v_4)$ the exotic neutralinos may be the lightest neutralinos, but have nevertheless masses of the order $m_{Z'}$ even for small M'.
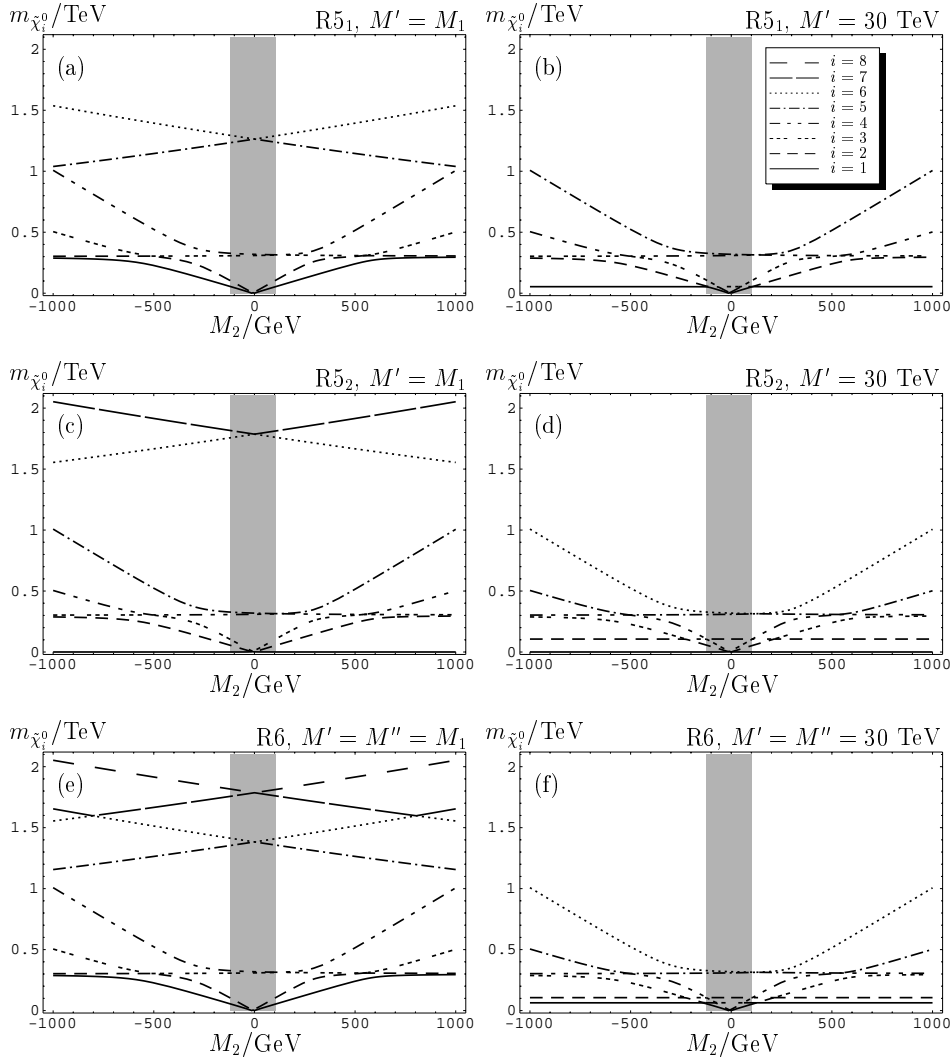
Fig. 1a–f. Mass spectra of the neutralinos in the E$_6$ models for $\tan\beta = 5$, $\lambda = 0.1$, v_3 [$= v_4$] $= 3$ TeV and $M_1 = M_2\, 5/3 \tan^2\theta_W$: **a** model R5$_1$ with $M' = M_1$, **b** model R5$_1$ with $M' = 30$ TeV, **c** model R5$_2$ with $M' = M_1$, **d** model R5$_2$ with $M' = 30$ TeV, **e** model R6 with $M' = M'' = M_1$ and **f** model R6 with $M' = M'' = 30$ TeV. The shaded area marks the experimentally excluded parameter space [28]

In model R5$_2$ the exotic 3×3 submatrix is singular so the lightest neutralino is very light ($m_{\tilde\chi_1^0} = 0.2$ GeV) and has mainly singlino character (Fig. 1c). Nevertheless the exotic neutralinos decouple from the MSSM-like ones in good approximation [26].

If the GUT relation for M' is relaxed in model R5$_1$ and a large value $M' \gg v_3$ chosen, a LSP with singlino character is possible [22]. Then a mechanism like the see-saw effect in the neutrino mass matrix [27] occurs in the submatrix of the exotic neutralinos which results in a light singlino-like and a very heavy $\tilde Z'$-gaugino-like neutralino (the $\tilde\chi_6^0$) with masses

$$m_{\text{singlino-like}} \approx Y_3'^2 g'^2 \frac{v_3^2}{2M'}, \quad m_{\tilde\chi_6^0} \approx M'. \quad (10)$$

Fig. 1b shows the neutralino mass spectrum for $M' = 30$ TeV resulting in a singlino-like neutralino with mass 54 GeV, which is the LSP in a large fraction of the parameter space.

In model R5$_2$ a large value of $M' = 30$ TeV leads to a second light singlino-like neutralino with mass 106 GeV (Fig. 1d). In model R6 a large value for M' and GUT relation $M'' = M_1$ and vice versa results again in one

light singlino, whereas for large values of both M' and M'' two light singlinos are possible due to this see-saw effect in the 4×4 submatrix of the exotic neutralinos. This is shown in Fig. 1f for $M' = M'' = 30$ TeV with singlino-like neutralinos of masses 64 GeV and 106 GeV which are the lightest and second lightest neutralino in large parameter regions.

To conclude because of the structure of the neutralino mixing matrix in the considered E$_6$ inspired models the lighter four neutralinos have MSSM-like character in models R5$_1$ and R6 with $M_1, M_2, M', M'', \lambda v_3 = \mathcal{O}(100 \text{ GeV})$. Light singlino-like neutralinos exist in the model R5$_2$ and the models R5$_1$ and R6 with large M', M'', whereas light neutralinos never have dominant Z' gaugino character.

3 Neutralino production at an electron-positron collider

3.1 Cross sections

The production of neutralinos $e^+e^- \to \tilde\chi_i^0 \tilde\chi_j^0$ in the E$_6$ models proceeds via s channel exchange of the neutral

gauge bosons Z_n and t and u channel exchange of selectrons. The production cross section

$$\sigma = (\sigma_Z + \sigma_{\tilde{e}} + \sigma_{Z\tilde{e}}) \frac{1}{2}(2 - \delta_{ij}) \qquad (11)$$

is derived from the Lagrangians in App. F.

For beams with longitudinal polarization P_3^- for electrons and P_3^+ for positrons ($-1 < P_3^\mp < 1$) one obtains for the s channel contribution

$$
\begin{aligned}
\sigma_Z =& \frac{g_2^4}{32\pi \cos^4\theta_W} \frac{w_{ij}}{s^2} \Bigg\{ \sum_{n=1}^{n_Z} \Big[|D_{Z_n}(s)|^2 (O_{ij}^{''nL})^2 \\
&\times \big[(1 - P_3^- P_3^+)(L_n^2 + R_n^2) + (P_3^- - P_3^+)(R_n^2 - L_n^2)\big] \Big] \\
&+ \sum_{\substack{n,n'=1\\n<n'}}^{n_Z} \Big[2\,\mathrm{Re}[D_{Z_n}(s)D_{Z_{n'}}^*(s)] O_{ij}^{''nL} O_{ij}^{''n'L} \\
&\times \big[(1 - P_3^- P_3^+)(L_n L_{n'} + R_n R_{n'}) \\
&+ (P_3^- - P_3^+)(R_n R_{n'} - L_n L_{n'})\big] \Big] \Bigg\} \\
&\times \Big\{ s^2 - (m_{\tilde{\chi}_i^0}^2 - m_{\tilde{\chi}_j^0}^2)^2 + \frac{1}{3}w_{ij}^2 - 4\eta_{\tilde{\chi}_i^0}\eta_{\tilde{\chi}_j^0} m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} s \Big\}.
\end{aligned}
\qquad (12)
$$

The t and u channel terms read

$$
\begin{aligned}
\sigma_{\tilde{e}} =& \frac{g_2^4}{32\pi} \frac{w_{ij}}{s^2} \Bigg\{ (f_{ei}^L)^2 (f_{ej}^L)^2 \big[(1 - P_3^- P_3^+) - (P_3^- - P_3^+)\big] \\
&\times \Bigg[\frac{s^2 - (m_{\tilde{\chi}_i^0}^2 - m_{\tilde{\chi}_j^0}^2)^2 - 4s^2 d_L(1 - d_L)}{4s^2 d_L^2 - w_{ij}^2} + 1 \\
&+ \frac{2sd_L - s + \eta_{\tilde{\chi}_i^0}\eta_{\tilde{\chi}_j^0} m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0}/d_L}{w_{ij}} \ln\left|\frac{2sd_L - w_{ij}}{2sd_L + w_{ij}}\right| \Bigg] \Bigg\} \\
&+ (L \to R, \quad P_3^- \to P_3^+)
\end{aligned}
\qquad (13)
$$

and the interference terms are

$$
\begin{aligned}
\sigma_{Z\tilde{e}} =& \frac{g_2^4}{32\pi \cos^2\theta_W} \frac{w_{ij}}{s^2} \Bigg\{ \sum_{n=1}^{n_Z} \Big[\mathrm{Re}(D_{Z_n}(s)) O_{ij}^{''nL} L_n \Big] \\
&\times f_{ei}^L f_{ej}^L \big[(1 - P_3^- P_3^+) - (P_3^- - P_3^+)\big] \\
&\times \Bigg[\Big\{ \Big(s^2 - (m_{\tilde{\chi}_i^0}^2 - m_{\tilde{\chi}_j^0}^2)^2 - 4\eta_{\tilde{\chi}_i^0}\eta_{\tilde{\chi}_j^0} m_{\tilde{\chi}_i^0} m_{\tilde{\chi}_j^0} s \\
&- 4s^2 d_L(1 - d_L) \Big) \Big/ \big(w_{ij}\big) \Big\} \ln\left|\frac{2sd_L - w_{ij}}{2sd_L + w_{ij}}\right| \\
&- 4s(1 - d_L) \Bigg] \Bigg\} - (L \to R, \quad P_3^- \to P_3^+).
\end{aligned}
\qquad (14)
$$

The following abbreviations have been used

$$D_{Z_n}(s) \equiv \frac{1}{s - m_{Z_n}^2 + i m_{Z_n}\Gamma_{Z_n}}, \qquad (15)$$

$$w_{ij} \equiv \Big[s - (m_{\tilde{\chi}_i^0} + m_{\tilde{\chi}_j^0})^2 \Big]^{\frac{1}{2}}$$

Table 2. Parameters of the neutralino mixing scenarios in the E_6 models and in the MSSM with $\mu = -\lambda v_3$

Scenario	H	G	M
M_2/GeV	400	-209	-251
λ	0.037	0.133	0.058
M_1		$M_2\, 5/3 \tan^2\theta_W$	
$\tan\beta$		5	
$v_3\ [= v_4]$/GeV		3000	

$$\times \Big[s - (m_{\tilde{\chi}_i^0} - m_{\tilde{\chi}_j^0})^2 \Big]^{\frac{1}{2}}, \qquad (16)$$

$$d_{L,R} \equiv \frac{1}{2s}\Big(s + 2m_{\tilde{e}_{L,R}}^2 - m_{\tilde{\chi}_i^0}^2 - m_{\tilde{\chi}_j^0}^2 \Big). \qquad (17)$$

For $n_Z = 1$ one recovers the cross section of the MSSM [29], if the couplings $O_{ij}^{''1L} \equiv O_{ij}^{''L}$, $L_1 \equiv L$, $R_1 \equiv R$ and $f_{ei}^{L/R}$ are interpreted as those of the MSSM. In models R5$_1$ and R5$_2$ the number of neutral gauge bosons is $n_Z = 2$, in model R6 $n_Z = 3$. Note that all couplings are assumed to be real due to CP conservation.

Finally we define the polarization asymmetry

$$A_{\mathrm{LR}} = \frac{\sigma(-P_3^-, P_3^+) - \sigma(+P_3^-, P_3^+)}{\sigma(-P_3^-, P_3^+) + \sigma(+P_3^-, P_3^+)}. \qquad (18)$$

with respect to the electron polarization P_3^- and fixed positron polarization P_3^+. In the following numerical discussions of A_{LR} we use $P_3^- = 0.85$ [30].

3.2 Scenarios

Neutralino production will be discussed in representative mixing scenarios in the extended models R5$_1$, R5$_2$ and R6 (see Tables 2–5).

The experimental lower mass bounds on the new E_6 gauge bosons of about 600 GeV [31] are respected by choosing a value of 3000 GeV for the vacuum expectation values of the singlet fields v_3 and v_4 which leads to $m_{Z_2} = 1264$ GeV in the model R5$_1$, $m_{Z_2} = 1786$ GeV in R5$_2$ and $m_{Z_2} = 1383$ GeV, $m_{Z_3} = 1786$ GeV in R6. The widths of the new gauge bosons are estimated by $\Gamma_{Z_{2,3}} \approx 0.014\, m_{Z_{2,3}}$ [25].

The neutralino mixing parameters M_2, λ, $\tan\beta = v_2/v_1$, v_3 [and v_4] are fixed in Table 2 for three scenarios H, G, M where the light MSSM-like neutralinos have higgsino, gaugino and mix character, respectively, and the mass of the lightest MSSM-like neutralino is 100 GeV. The higgsino mass parameter μ of the MSSM is recovered by $\mu = -\lambda v_3$. Since the neutralino production cross sections depend only weakly on $\tan\beta$, we confine ourselves to one value $\tan\beta = 5$.

The neutralino masses and mixings of Table 3 are obtained assuming the GUT relation [15,18]

$$M_1 = M'\ [= M''] = M_2 \frac{5}{3}\tan^2\theta_W. \qquad (19)$$

Table 3. Neutralino masses and mixings in the MSSM with $\mu = -\lambda v_3$ and in the E$_6$ models R5$_1$ and R6 with $M' [= M''] = M_1$

Model	MSSM, R5$_1$, R6 ($M' [= M''] = M_1$)		
Scenario	H	G	M
$m_{\tilde{\chi}_1^0}$/GeV	100	100	100
$\tilde{\chi}_1^0$-character	higgsino	gaugino	mix
$m_{\tilde{\chi}_2^0}$/GeV	124	192	161
$\tilde{\chi}_2^0$-character	higgsino	gaugino	mix

Table 4. Neutralino masses and mixings in the E$_6$ models R5$_1$ with $M' = 20$ TeV and R6 with $M' = 32$ TeV, $M'' = M_1$

Model	R5$_1$ ($M' = 20$ TeV), R6 ($M' = 32$ TeV)		
Scenario	H	G	M
$m_{\tilde{\chi}_1^0}$/GeV	80	80	80
$\tilde{\chi}_1^0$-character	singlino	singlino	singlino
$m_{\tilde{\chi}_2^0}$/GeV	100	100	100
$\tilde{\chi}_2^0$-character	higgsino	gaugino	mix

Table 5. Neutralino masses and mixings in the E$_6$ model R5$_2$ with $M' = M_1$

Model	R5$_2$ ($M' = M_1$)		
Scenario	H	G	M
$m_{\tilde{\chi}_1^0}$/GeV	0.1	0.2	0.1
$\tilde{\chi}_1^0$-character	singlino	singlino	singlino
$m_{\tilde{\chi}_2^0}$/GeV	100	100	100
$\tilde{\chi}_2^0$-character	higgsino	gaugino	mix

Then the light neutralinos are MSSM-like in models R5$_1$ and R6.

In Table 4, however, we abandon the GUT-relation for M' and choose large values which lead to light neutralinos with singlino character [22, 24]. Here a singlino-like $\tilde{\chi}_1^0$ with mass of about 80 GeV appears in models R5$_1$ and R6. The $\tilde{\chi}_1^0$ in the model R5$_2$ is always very light with mass $\mathcal{O}(0.1$ GeV$)$ as shown in Table 5.

The neutralino cross sections in e^+e^- annihilation also depend on the masses of the left and right selectrons. In order to compare the results, we use the same weak scale selectron masses throughout our numerical analysis. First the mass of the left selectron is fixed at $m_{\tilde{e}_L} = 300$ GeV in both the MSSM and the E$_6$ models. Then one obtains a right selectron mass $m_{\tilde{e}_R} = 200$ GeV by mSUGRA-type renormalization group equations with parameters $M_2 = 300$ GeV and $m_0 = 132$ GeV in the MSSM [32]. Assuming $\tilde{M}_{\tilde{e}_L}^2 = \tilde{M}_{\tilde{e}_R}^2$ in the selectron mass formulas in the E$_6$ models (App. E) the large D-terms of the new U(1) gauge factors result in $m_{\tilde{e}_R} = 753$ GeV in R5$_1$ and $m_{\tilde{e}_R} = 1022$ GeV in R5$_2$ and R6. Note that in the E$_6$ models in contrast to the MSSM the right selectrons are much heavier than the left ones [10, 33].

In the scenarios without GUT relations we keep the above values for the selectron masses as free parameters. Otherwise the RGE would induce weak scale selectron masses of the order of M' which strongly suppress the cross sections especially in gaugino scenarios.

3.3 Numerical results

3.3.1 $\tilde{\chi}_1^0$ and $\tilde{\chi}_2^0$ MSSM-like

In the models R5$_1$ and R6 with $M' [= M''] = M_1$ both light neutralinos $\tilde{\chi}_1^0$ and $\tilde{\chi}_2^0$ have MSSM-like character in all scenarios of Table 2 with masses given in Table 3. The total cross sections for neutralino production $e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$ with unpolarized beams ($P_3^- = P_3^+ = 0$) and the polarization asymmetries for $P_3^+ = 0$ are shown in Fig. 2.

In scenario H with higgsino-like neutralinos the new gauge bosons cause high narrow resonances, whereas otherwise the cross sections are similar to the MSSM. The polarization asymmetries, however, show a much wider resonance effect. Contrary to the MSSM, where the electron polarization asymmetry $A_{\mathrm{LR}} \sim 0.1$ is nearly independent of the beam energy, it changes sign at about 800 GeV in model R5$_1$ and at about 650 GeV in model R6.

We do not explicitly show polarization asymmetries for $P_3^+ \neq 0$ in scenario H since additional polarization of the positron beam only shifts the asymmetry in all models for $P_3^+ > 0$ to higher or for $P_3^+ < 0$ to lower values. In the E$_6$ models one obtains at threshold $A_{\mathrm{LR}} \sim 0.6$ for $P_3^+ = +0.6$ and $A_{\mathrm{LR}} \sim -0.4$ for $P_3^+ = -0.6$, the change of sign of A_{LR} occurs for $P_3^+ = +0.6$ at $\sqrt{s} = 1150$ GeV (R5$_1$) or $\sqrt{s} = 1050$ GeV (R6) and for $P_3^+ = -0.6$ at the Z' resonance significantly above the energy range of a linear collider at first stage.

In scenario G the gaugino-like light neutralinos are mainly produced by the exchange of left selectrons leading to obviously smaller gauge boson resonances. Choosing the same left selectron mass in the MSSM and the E$_6$ models the cross sections are rather similar. The different masses of the right selectrons, however, lead to distinct differences between the electron polarization asymmetries that are largest just above threshold [34] where $A_{\mathrm{LR}} \sim 0.84$ in the E$_6$ models and $A_{\mathrm{LR}} \sim 0.59$ in the MSSM.

Positron polarization hardly affects the polarization asymmetries in the E$_6$ models whereas the asymmetry in the MSSM is shifted to lower (higher) values for $P_3^+ < 0$ (> 0). Figure 3 shows how negative positron beam polarization $P_3^+ = -0.6$ enhances the differences between the models ($A_{\mathrm{LR}} \sim 0.8$ at threshold in the E$_6$ models compared to $A_{\mathrm{LR}} \sim 0.15$ in the MSSM).

In scenario M the gaugino content of the light neutralinos $\tilde{\chi}_1^0$ and $\tilde{\chi}_2^0$ couples mainly to right selectrons which are much heavier in the E$_6$ models than in the MSSM. The different right selectron masses lead to significantly smaller cross sections in the E$_6$ models below the resonances compared to the MSSM and clearly distinguishable electron polarization asymmetries at threshold ($A_{\mathrm{LR}} \sim 0.4$ in the model R6, 0.2 in the model R5$_1$ and -0.75 in the MSSM).

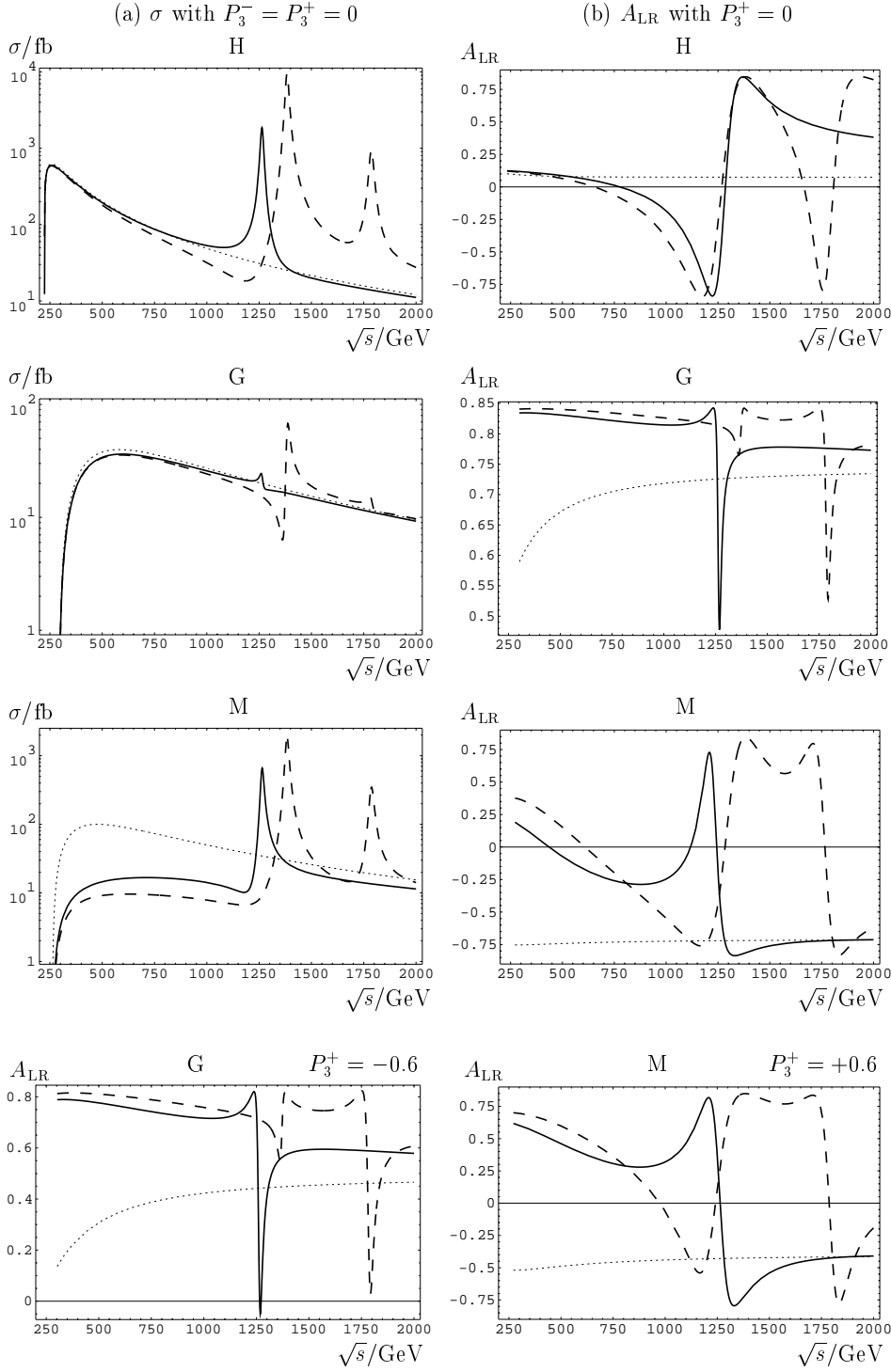(a) σ with $P_3^- = P_3^+ = 0$ (b) A_{LR} with $P_3^+ = 0$

Fig. 2. **a** Total cross sections for $P_3^- = P_3^+ = 0$ and **b** polarization asymmetries for $P_3^+ = 0$ of the process $e^+e^- \to \tilde{\chi}_1^0 \tilde{\chi}_2^0$ in the scenarios of Table 2 with $M'[= M''] = M_1$ in the models R5$_1$ (solid), R6 (dashed) and the MSSM (dotted)

Fig. 3. Polarization asymmetries for polarized positron beam of the process $e^+e^- \to \tilde{\chi}_1^0 \tilde{\chi}_2^0$ in the scenarios G and M of Table 2 with $M'[= M''] = M_1$ in the models R5$_1$ (solid), R6 (dashed) and the MSSM (dotted)

This effect can be used to determine the selectron masses [35].

Due to the higgsino content of the neutralinos, the cross sections show high narrow resonances as in scenario H with wider resonances in A_{LR} than in scenario G. Positive positron beam polarization shifts the asymmetries to higher values in all models but in a larger extend in the E$_6$ models than in the MSSM (see Fig. 3). The threshold values $A_{\mathrm{LR}} \sim 0.7$ in model R6, 0.6 in model R5$_1$ and -0.5

in the MSSM for $P_3^+ = +0.6$ offer a clear signature to distinguish between the models.

3.3.2 $\tilde{\chi}_1^0$ singlino-like and $\tilde{\chi}_2^0$ MSSM-like

In the models R5$_1$ with $M' = 20$ TeV, R5$_2$ with $M' = M_1$ and R6 with $M' = 32$ TeV the lightest neutralino $\tilde{\chi}_1^0$ is a nearly pure singlino. Then the second lightest neutralino
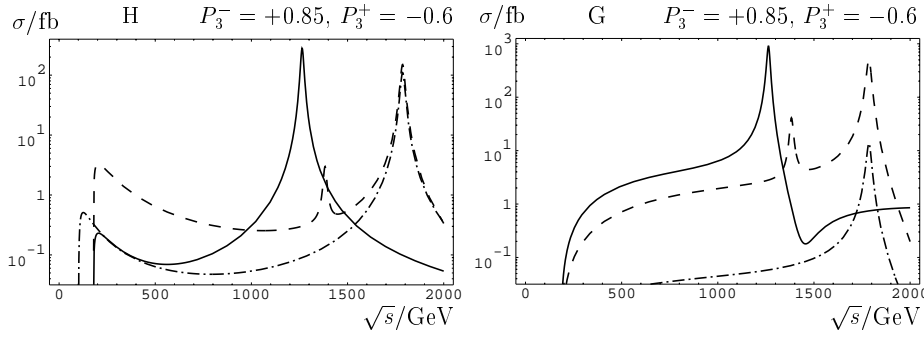
Fig. 4. Total cross sections for e^+e^- $\rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$ in the scenarios H and G of Table 2 with beam polarization $P_3^- = +0.85$, $P_3^+ = -0.6$ in the models R5$_1$ with $M' = 20$ TeV (solid), R5$_2$ with $M' = M_1$ (dashed-dotted) and R6 with $M' = 32$ TeV (dashed)
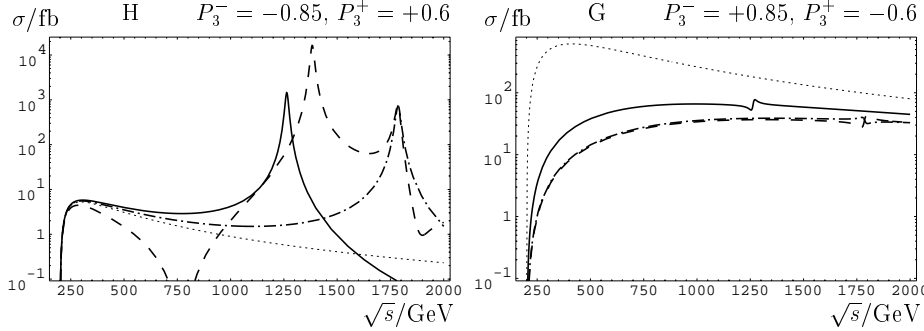
Fig. 5. Total cross sections for e^+e^- $\rightarrow \tilde{\chi}_2^0\tilde{\chi}_2^0$ in scenario H with beam polarization $P_3^- = -0.85$, $P_3^+ = +0.6$ and scenario G with $P_3^- = +0.85$ and $P_3^+ = -0.6$ in the models R5$_1$ with $M' = 20$ TeV (solid), R5$_2$ with $M' = M_1$ (dashed-dotted) and R6 with $M' = 32$ TeV (dashed). For comparison the corresponding invisible process $e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0$ in the MSSM is plotted (dotted)

has similar higgsino, gaugino or mix character in the scenarios H, G or M, respectively, as the $\tilde{\chi}_1^0$ in the MSSM (Tables 4 and 5). Since the singlino content of the neutralinos does not couple to selectrons and the standard gauge boson the direct production of singlino dominated neutralinos is generally suppressed. Nevertheless Fig. 4 shows total cross sections for a beam polarization $P_3^- = +0.85$ and $P_3^+ = -0.6$ that reach some 0.1 fb outside the gauge boson resonances in all models in scenario H and in models R5$_1$ and R6 in scenario G. So even the direct production of singlino-like neutralinos may be detectable in a considerable domain of the parameter space at a linear collider with a luminosity of 500 fb^{-1} at $\sqrt{s} = 500$ GeV [36,37].

The size of the cross sections in the different models mainly depends on the MSSM-components of the singlino dominated $\tilde{\chi}_1^0$. In scenario H the doublet higgsino content $(N_{13}^2 + N_{14}^2)$, which couples to the doublet higgsino-like $\tilde{\chi}_2^0$, is 0.22 % in the model R5$_1$, 0.17 % in R5$_2$ and 0.42 % in R6. For $\sqrt{s} \lesssim 1$ TeV the polarized cross sections with $P_3^- = +0.85$ and $P_3^+ = -0.6$ are about 1.5 to 3 times larger than the unpolarized cross sections depending on the model.

In scenario G the singlino dominated $\tilde{\chi}_1^0$ has a MSSM-gaugino content $(N_{11}^2 + N_{12}^2)$ of 1.5 % in R5$_1$, 0.05 % in R5$_2$ and 1.7 % in R6. Therefore the cross section in R5$_2$ is smaller than 0.1 fb outside the resonance while the larger cross section in R5$_1$ compared to R6 is caused by the smaller mass of the right selectron. The beam polarization enhances the unpolarized cross sections by a factor of about 3.

The cross sections for the pair production of the lightest MSSM-like neutralinos $e^+e^- \rightarrow \tilde{\chi}_2^0\tilde{\chi}_2^0$ are plotted in Fig. 5. The corresponding process $e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0$ in the MSSM is invisible.

In scenario H a beam polarization $P_3^- = -0.85$, $P_3^+ = +0.6$ roughly doubles the unpolarized cross sections to about 4 to 6 fb for $\sqrt{s} \lesssim 1$ TeV. Pair production of higgsino-like neutralinos is generally suppressed compared to the associated production of higgsino dominated neutralinos in Fig. 2. The minimum of the cross section in model R6 at $\sqrt{s} \approx 750$ GeV is caused by negative interference effects between the contributions of the three gauge bosons.

In scenario G the opposite beam polarization $P_3^- = +0.85$ and $P_3^+ = -0.6$ leads to a maximum enhancement of the unpolarized cross sections by a factor between 2 and 3. Since the gaugino-like $\tilde{\chi}_2^0$ couples mainly to right selectrons in the whole parameter space also the cross section for the pair production of gaugino-like neutralinos is suppressed in the E_6 models compared to the MSSM.

The polarized cross sections in scenarios H and G are clearly above the discovery limit of a high luminosity linear collider. An even more distinctive signal can be expected from the process $e^+e^- \rightarrow \tilde{\chi}_2^0\tilde{\chi}_3^0$ with a cross section similar to the cross section of $e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$ for MSSM-like $\tilde{\chi}_1^0$ and $\tilde{\chi}_2^0$ (see Fig. 2).

4 Conclusion

We have analyzed the neutralino sector in E_6 inspired extended supersymmetric models with additional neutral gauge bosons and singlet Higgs fields. To obey the experimental lower mass bounds of the new gauge bosons the vacuum expectation values of the singlet fields must be of the order of some TeV.

In a rank-5 model with two singlets the lightest neutralino is always a very light nearly pure singlino. Light

neutralinos with singlino character also appear in a rank-5 model with one singlet and in the rank-6 model if the U(1)$'$ (U(1)$''$) gaugino mass parameter M' (M'') takes large values $\mathcal{O}(10\ \mathrm{TeV})$ because of a see-saw-like mechanism in the submatrix of the exotic neutralinos. Two light singlino-like neutralinos may exist in the rank-5 model with two singlets for large M' and in the rank-6 model for both M' and M'' large. However, light neutralinos in the discussed E$_6$ models never have dominant Z' (Z'') gaugino character. Assuming the GUT relation for the gaugino mass parameters in the rank-5 model with one singlet and the rank-6 model, the MSSM-like neutralinos decouple and the masses of the exotic neutralinos are of the order of the singlet vacuum expectation values in the TeV range.

The production of neutralinos in e^+e^- annihilation proceeds via s channel exchange of neutral gauge bosons and t and u channel exchange of selectrons. The production cross sections of neutralinos with dominant higgsino character show narrow high resonances of the new gauge bosons but are otherwise rather similar to the MSSM. If the resonances of the new gauge bosons are not accessible at the first stage of a linear collider, the use of polarized beams is an important tool to discriminate between the MSSM and the E$_6$ models since the polarization asymmetries show significantly wider resonance effects far below the mass of the new gauge bosons.

Assuming mSUGRA-type RGEs the models also differ by the selectron masses. Due to additional D-terms in the superpotential the right selectrons are much heavier than the left ones in the E$_6$ models contrary to the MSSM. Then in scenarios with large gaugino content of the neutralinos the differences between the models depend on the neutralino couplings to left and right selectrons. Especially if one or both selectrons cannot be produced directly at the first stage of a linear collider the determination of the selectron masses by measuring neutralino production cross sections offers a particularly suitable possibility to distinguish between the models. Polarization asymmetries show even more distinctive effects of the selectron masses.

The cross sections for the direct production of light singlino-like neutralinos are typically of the order of some fb outside the gauge boson resonances which is sufficient to be detected at a high luminosity linear collider. If the lightest supersymmetric particle is a singlino-like neutralino, pair production of $\tilde{\chi}_2^0$ with a significant gaugino content is visible while pair production of a higgsino-dominated neutralino is generally suppressed. Again, beam polarization enhances the cross sections by a factor up to 3 and improves the discovery chances at a linear collider.

Appendix: E$_6$ inspired models

A Symmetry breaking

The exceptional group E$_6$ may be a suitable candidate for a gauge group of a Grand Unified Theory (GUT). The rank-6 group E$_6$ is a natural extension of the rank-4 group SU(5) and the rank-5 group SO(10) and contains the maximal subgroup SO(10) $\times$ U(1). E$_6$ holds complex representations necessary to describe chiral fermions and is naturally free of anomalies. Furthermore the compactification of an E$_8 \times$ E$_8'$ string theory to four dimensions can lead to E$_6$ as an effective GUT group [8,14,38,39].

In order to get a low energy gauge group of the form SU(3) $\times$ SU(2) $\times$ U(1)^n the E$_6$ group has to be broken [8]. If E$_6$ is broken directly to a low energy group of rank 5 these group

$$\mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\eta} \qquad (A.1)$$

is uniquely determined. If E$_6$ is broken to a low energy group of rank 6 several possibilities arise. We confine ourselves on the case of two additional U(1) factors in comparison to the SM

$$\mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\psi \times U(1)_\chi}, \qquad (A.2)$$

where U(1)$_\psi$ and U(1)$_\chi$ are defined by

$$\mathrm{E_6 \to SO(10) \times U(1)_\psi}, \qquad \mathrm{SO(10) \to SU(5) \times U(1)_\chi}. \qquad (A.3)$$

For suitable large vacuum expectation values of the symmetry breaking Higgs fields the rank-6 model can be reduced to an "effective" rank-5 model (U(1)$_\psi \times$ U(1)$_\chi \to$ U(1)$_\theta$), where one new gauge boson decouples from low energy theory [8,39]. Then the remaining new gauge boson $Z' = Z_\psi \cos\theta - Z_\chi \sin\theta$ is in general a mixture of Z_ψ and Z_χ. For $\theta = \arcsin\sqrt{3/8}$ the quantum numbers of the true rank-5 model ("model η") are recovered. In this paper we focus on this model η in the rank-5 case. $\theta = -\arctan\sqrt{1/15}$ gives the so called U(1)$_N$ model which is favored by neutrino phenomenology and leptogenesis considerations [40].

B Particle content

Each chiral generation of fermions belongs to a fundamental representation **27** of E$_6$ which decomposes according to

$$\mathbf{27} = (\mathbf{16}, \mathbf{10}) + (\mathbf{16}, \bar{\mathbf{5}}) + (\mathbf{16}, \mathbf{1}) + (\mathbf{10}, \mathbf{5}) + (\mathbf{10}, \bar{\mathbf{5}}) + (\mathbf{1}, \mathbf{1})$$
$$(B.1)$$

in terms of the subgroups SO(10) and SU(5) of E$_6$ [8]. Table 6 shows this for one generation of matter in the E$_6$ models. In order to fill the **27** new "exotic" fields are

Table 6. Fermionic particle content of the fundamental **27** representation of E_6, assignment of the fermions to the subgroups SO(10), SU(5) and SU(3)$_C$ and quantum numbers according to SU(2)$_L$, U(1)$_Y$, U(1)$_\eta$, U(1)$_\psi$, U(1)$_\chi$ [8]

		SU(3)$_C$	T_{3L}	Y	Y_η	Y_ψ	Y_χ
$Q \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L$	$(\mathbf{16},\mathbf{10})$	$\mathbf{3}$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$	$1/3$	$2/3$	$\sqrt{10}/6$	$-1/\sqrt{6}$
u_L^c		$\bar{\mathbf{3}}$	0	$-4/3$	$2/3$	$\sqrt{10}/6$	$-1/\sqrt{6}$
e_L^c		$\mathbf{1}$	0	2	$2/3$	$\sqrt{10}/6$	$-1/\sqrt{6}$
$L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L$	$(\mathbf{16},\bar{\mathbf{5}})$	$\mathbf{1}$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$	-1	$-1/3$	$\sqrt{10}/6$	$3/\sqrt{6}$
d_L^c		$\bar{\mathbf{3}}$	0	$2/3$	$-1/3$	$\sqrt{10}/6$	$3/\sqrt{6}$
ν_L^c	$(\mathbf{16},\mathbf{1})$	$\mathbf{1}$	0	0	$5/3$	$\sqrt{10}/6$	$-5/\sqrt{6}$
$H \equiv \begin{pmatrix} N \\ E \end{pmatrix}_L$	$(\mathbf{10},\bar{\mathbf{5}})$	$\mathbf{1}$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$	-1	$-1/3$	$-\sqrt{10}/3$	$-2/\sqrt{6}$
h_L^c		$\bar{\mathbf{3}}$	0	$2/3$	$-1/3$	$-\sqrt{10}/3$	$-2/\sqrt{6}$
$H^c \equiv \begin{pmatrix} E \\ N \end{pmatrix}_L^c$	$(\mathbf{10},\mathbf{5})$	$\mathbf{1}$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$	1	$-4/3$	$-\sqrt{10}/3$	$2/\sqrt{6}$
h_L		$\mathbf{3}$	0	$-2/3$	$-4/3$	$-\sqrt{10}/3$	$2/\sqrt{6}$
S_L^c	$(\mathbf{1},\mathbf{1})$	$\mathbf{1}$	0	0	$5/3$	$2\sqrt{10}/3$	0

necessary in comparison to the MSSM. The breaking of E_6 fixes the color, isospin and hypercharge of these "exotics" but not their baryon and lepton numbers and their R-parity. In this paper we assume vanishing baryon and lepton numbers and R-parity -1 for the fermions H, H^c, ν_L^c and S_L^c. So they can be interpreted as superpartners of doublet and singlet Higgs fields of the model.

It is always possible to choose a basis that only the Higgs fields of one generation get vacuum expectation values, conventionally the fields of the third generation. So the two doublet Higgs fields of the model H_1 and H_2 can be identified as

$$H_1 \equiv (\tilde{H})_3 \quad \text{and} \quad H_2 \equiv (\tilde{H}^c)_3 \qquad \text{(B.2)}$$

and two singlet Higgs fields as

$$N_1 \equiv (\tilde{S}_L^c)_3 \quad \text{and} \quad N_2 \equiv (\tilde{\nu}_L^c)_3 \qquad \text{(B.3)}$$

with the vacuum expectation values (vevs)

$$\langle H_1 \rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad \langle H_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad \langle N_1 \rangle = v_3, \quad \langle N_2 \rangle = v_4 \, . \qquad \text{(B.4)}$$

The corresponding fields of the first two generations which obtain no vevs are called "unHiggs" [8] and are discussed in detail in [41]. In particular the corresponding "unhiggsinos" do not mix with the ordinary neutralinos. However the mass of the lightest neutral unhiggsino has a strict upper bound of about 100 GeV, hence it may be the LSP in some areas of the parameter space [42]. This case has to be considered in the analysis of the neutralino decay signatures, but the results regarding the mass spectra and the production of neutralinos discussed in this paper remain valid.

In the case of the rank-5 models only one singlet vev is necessary to break the extended gauge group. If the second singlet obtains no vev it decouples from the neutralino sector and is also considered an unhiggsino (rank-5 model with one singlet, R5$_1$). This model also avoids the problems with the creation of a vev for the second singlet [8,15,19,43].

C Gauge boson sector

Models with one (rank-5 models) or two (rank-6 models) U(1) factors in the gauge group contain one (Z') or two (Z' and Z'') new neutral gauge bosons in comparison to the SM. These new gauge bosons mix with the standard Z boson Z^{SM} to form mass eigenstates Z_1, Z_2 [and Z_3].

In the rank-5 [rank-6] model with the electroweak gauge group

$$\mathrm{SU(2)_L \times U(1)}_Y \times \mathrm{U(1)}' \, [\times \mathrm{U(1)}''] \qquad \text{(C.1)}$$

the mass term of the gauge bosons without abelian gauge kinetic mixing reads

$$\mathcal{L}_M^{\mathrm{Gauge}} = \frac{1}{2} g_2^2 (v_1^2 + v_2^2) W_\mu^+ W^{-\mu}$$

$$+ \frac{1}{2} \left(A_\mu, Z_\mu^{\mathrm{SM}}, Z_\mu' \, [, Z_\mu''] \right) X \begin{pmatrix} A^\mu \\ Z^{\mathrm{SM}\mu} \\ Z'^\mu \\ [Z''^\mu] \end{pmatrix} . \qquad \text{(C.2)}$$

As in the SM $W^\pm$ denote the charged gauge bosons with mass

$$m_W^2 = \frac{1}{2} g_2^2 (v_1^2 + v_2^2). \qquad \text{(C.3)}$$

$$X = \frac{1}{2} \left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & (g_1^2 + g_2^2)(v_1^2 + v_2^2) & g' \sqrt{g_1^2 + g_2^2}(v_1^2 Y_1' - v_2^2 Y_2') & g'' \sqrt{g_1^2 + g_2^2}(v_1^2 Y_1'' - v_2^2 Y_2'') \\ 0 & g' \sqrt{g_1^2 + g_2^2}(v_1^2 Y_1' - v_2^2 Y_2') & g'^2 \sum_{i=1}^4 v_i^2 Y_i'^2 & g' g'' \sum_{i=1}^4 v_i^2 Y_i' Y_i'' \\ 0 & g'' \sqrt{g_1^2 + g_2^2}(v_1^2 Y_1'' - v_2^2 Y_2'') & g' g'' \sum_{i=1}^4 v_i^2 Y_i' Y_i'' & g''^2 \sum_{i=1}^4 v_i^2 Y_i''^2 \end{array} \right) \tag{C.4}$$

X is the mixing matrix of the neutral gauge bosons [44] (see (C.4) on top of the page) in the basis of the massless photon A and the two [three] massive gauge bosons Z^{SM}, Z' [and Z''] which mix. g_2, g_1, g' [and g''] denote the gauge couplings of the $\mathrm{SU(2)_L}$, $\mathrm{U(1)}_Y$, $\mathrm{U(1)}'$ [and $\mathrm{U(1)}''$] gauge factors, respectively. The Y_i' (Y_i'') are the $\mathrm{U(1)}'$ ($\mathrm{U(1)}''$) quantum numbers of the doublet and singlet Higgs fields and the v_i the respective vacuum expectation values according to (B.4).

In the rank-5 model the mixing matrix is obtained as the upper left 3×3 submatrix of (C.4) with the quantum numbers Y_i' fixed by (2). With $\tan \beta = v_2/v_1$ and

$$m_{Z^{\mathrm{SM}}}^2 \equiv \frac{m_W^2}{\cos^2 \theta_W} = \frac{1}{2}(g_1^2 + g_2^2)(v_1^2 + v_2^2) \tag{C.5}$$

the rank-5 mixing matrix becomes

$$X' = \left(\begin{array}{ccc} 0 & 0 & 0 \\ 0 & m_{Z^{\mathrm{SM}}}^2 & \delta m^2 \\ 0 & \delta m^2 & m_{Z'}^2 \end{array} \right) \tag{C.6}$$

with

$$\delta m^2 = m_{Z^{\mathrm{SM}}}^2 \frac{g'}{g_1} \sin \theta_W \left(Y_1' \cos^2 \beta - Y_2' \sin^2 \beta \right), \tag{C.7}$$

$$m_{Z'}^2 = \frac{1}{2} Y_3'^2 g'^2 v_3^2 + \frac{1}{2} Y_4'^2 g'^2 v_4^2 + m_{Z^{\mathrm{SM}}}^2 \frac{g'^2}{g_1^2}$$
$$\times \sin^2 \theta_W \left(Y_1'^2 \cos^2 \beta + Y_2'^2 \sin^2 \beta \right). \tag{C.8}$$

Thus the mass eigenstates Z_1 and Z_2 of the massive neutral gauge bosons are

$$\left(\begin{array}{c} Z_{1,\mu} \\ Z_{2,\mu} \end{array} \right) = N^Z \left(\begin{array}{c} Z_\mu^{\mathrm{SM}} \\ Z_\mu' \end{array} \right) \tag{C.9}$$

with the orthogonal 2×2 diagonalization matrix

$$N^Z = \left(\begin{array}{cc} \cos \delta & \sin \delta \\ -\sin \delta & \cos \delta \end{array} \right), \tag{C.10}$$

the mixing angle δ,

$$\tan \delta = \frac{\delta m^2}{m_{Z^{\mathrm{SM}}}^2 - m_{Z_2}^2}, \tag{C.11}$$

and the mass eigenvalues

$$m_{Z_1, Z_2}^2 = \frac{1}{2} \left(m_{Z^{\mathrm{SM}}}^2 + m_{Z'}^2 \mp \sqrt{(m_{Z^{\mathrm{SM}}}^2 - m_{Z'}^2)^2 + 4\delta m^4} \right). \tag{C.12}$$

In the rank-6 model with the full 4×4 mixing matrix (C.4) the quantum numbers of the doublet and singlet Higgs fields Y_i' and Y_i'' are fixed by (3). The submatrix of the massive gauge bosons Z^{SM}, Z' and Z'' can be diagonalized by an orthogonal 3×3 matrix N^Z

$$\left(\begin{array}{cccc} 0 & & & \\ & m_{Z_1} & & \\ & & m_{Z_2} & \\ & & & m_{Z_3} \end{array} \right) = \left(\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & N^Z & \\ 0 & & & \end{array} \right) X \left(\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & (N^Z)^\dagger & \\ 0 & & & \end{array} \right) \tag{C.13}$$

with the mass eigenstates Z_1, Z_2 and Z_3.

D Chargino sector

The chargino mass term in the Lagrangian

$$\mathcal{L}_{m_{\chi^\pm}} = -\frac{1}{2}(\psi^+, \psi^-) \left(\begin{array}{cc} 0 & X^T \\ X & 0 \end{array} \right) \left(\begin{array}{c} \psi^+ \\ \psi^- \end{array} \right) + \mathrm{h.c.} \tag{D.1}$$

with the chargino mass matrix

$$X = \left(\begin{array}{cc} M_2 & \sqrt{2}\, m_W \sin \beta \\ \sqrt{2}\, m_W \cos \beta & -\lambda v_3 \end{array} \right) \tag{D.2}$$

is the same as in the MSSM if $-\lambda v_3$ is identified as the MSSM parameter μ [1, 25, 45].

E Sfermion sector

The mass terms of the sfermions can be derived from the scalar potential [14]. Because of the spontaneous breaking of the new $\mathrm{U(1)}$ factors additional D-terms appear in comparison to the MSSM [4, 8, 10, 14, 21, 25, 33, 46]. Neglecting the mixing of the left and right sfermions which is small for the first two generations the mass terms in the rank-5 [rank-6] model are

$$m_{\tilde{f}_L}^2 = \tilde{M}_{\tilde{f}_L}^2 + m_f^2 + L\, m_{Z^{\mathrm{SM}}}^2 \cos 2\beta$$
$$+ Y'(f_L)\, \tilde{m}_{D'}^2 \left[+ Y''(f_L)\, \tilde{m}_{D''}^2 \right], \tag{E.1}$$

$$m_{\tilde{f}_R}^2 = \tilde{M}_{\tilde{f}_R}^2 + m_f^2 - R\, m_{Z^{\mathrm{SM}}}^2 \cos 2\beta$$
$$- Y'(f_R)\, \tilde{m}_{D'}^2 \left[- Y''(f_R)\, \tilde{m}_{D''}^2 \right]. \tag{E.2}$$

$\tilde{M}_{\tilde{f}_{L,R}}$ are the scalar mass parameters and

$$L = T_{3L} - Q \sin^2 \theta_W, \quad R = -Q \sin^2 \theta_W. \tag{E.3}$$

In the rank-5 model with one singlet [two singlets] the quantum numbers $Y'(f_{L,R})$ of the fermion fields are

$$Y'(f_L) = Y_\eta(f_L)\,, \quad Y'(f_R) = Y_\eta(f_R) = -Y_\eta(f_L^c) \quad \text{(E.4)}$$

as listed in Table 6 and the D-term is

$$\tilde{m}_{D'}^2 = \frac{1}{4}g'^2\left(Y_1'v_1^2 + Y_2'v_2^2 + Y_3'v_3^2\,[+Y_4'v_4^2]\right) \qquad \text{(E.5)}$$

with Y_i' according to (2).

In the rank-6 model the quantum numbers are

$$Y'(f_{L,R}) = Y_\psi(f_{L,R})\,, \qquad Y''(f_{L,R}) = Y_\chi(f_{L,R}) \quad \text{(E.6)}$$

as shown in Table 6 and the two new D-terms read

$$\tilde{m}_{D'}^2 = \frac{1}{4}g'^2\left(Y_1'v_1^2 + Y_2'v_2^2 + Y_3'v_3^2 + Y_4'v_4^2\right)\,, \quad \text{(E.7)}$$

$$\tilde{m}_{D''}^2 = \frac{1}{4}g''^2\left(Y_1''v_1^2 + Y_2''v_2^2 + Y_3''v_3^2 + Y_4''v_4^2\right) \quad \text{(E.8)}$$

with Y_i', Y_i'' according to (3).

F Lagrangians and couplings

F.1 Neutral currents Lagrangian

In the rank-5 [rank-6] model the neutral currents Lagrangian reads [8,39,47]

$$\mathcal{L}_{\text{NC}} = -eQ\bar{f}\gamma^\mu f A_\mu \qquad\qquad\qquad \text{(F.1)}$$
$$-\frac{g_2}{\cos\theta_W}\bar{f}_L(T_{3L} - Q\sin^2\theta_W)\gamma^\mu f_L Z_\mu^{SM}$$
$$-\frac{g_2}{\cos\theta_W}\bar{f}_R(-Q\sin^2\theta_W)\gamma^\mu f_R Z_\mu^{SM}$$
$$-g'\bar{f}_L\frac{Y'(f_L)}{2}\gamma^\mu f_L Z_\mu' - g'\bar{f}_R\frac{Y'(f_R)}{2}\gamma^\mu f_R Z_\mu'$$
$$\left[-g''\bar{f}_L\frac{Y''(f_L)}{2}\gamma^\mu f_L Z_\mu'' - g''\bar{f}_R\frac{Y''(f_R)}{2}\gamma^\mu f_R Z_\mu''\right].$$

Here $e \equiv g_2\sin\theta_W = \sqrt{4\pi\alpha}$ is the absolute value of the electron charge and f denotes the respective fermion field with $f_{L/R} = P_{L/R}f$ and the chiral projection operators $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$. The $Y'(f_{L/R})$ and $Y''(f_{L/R})$ are the U(1)$'$ and U(1)$''$ quantum numbers of the respective model as listed in Table 6.

In terms of the mass eigenstates of the neutral gauge bosons the Lagrangian has the form

$$\mathcal{L}_{\text{NC}} = -eQ\bar{f}\gamma^\mu f A_\mu$$
$$-\sum_{n=1}^{n_Z}\frac{g_2}{\cos\theta_W}\bar{f}\gamma^\mu\left[L_n P_L + R_n P_R\right]f Z_{n,\mu} \quad \text{(F.2)}$$

with (E.3) and

$$L_n = LN_{n1}^Z + \frac{Y'(f_L)}{2}\frac{g'}{g_1}\sin\theta_W N_{n2}^Z$$

$$\left[+\frac{Y''(f_L)}{2}\frac{g''}{g_1}\sin\theta_W N_{n3}^Z\right], \qquad \text{(F.3)}$$

$$R_n = RN_{n1}^Z + \frac{Y'(f_R)}{2}\frac{g'}{g_1}\sin\theta_W N_{n2}^Z$$

$$\left[+\frac{Y''(f_R)}{2}\frac{g''}{g_1}\sin\theta_W N_{n3}^Z\right]. \qquad \text{(F.4)}$$

In the rank-5 model with $n_Z = 2$ the couplings $Y'(f_{L,R})$ are given in (E.4) and N^Z in (C.10), whereas in the rank-6 model ($n_Z = 3$) $Y'(f_{L,R})$, $Y''(f_{L,R})$ according to (E.6) and N^Z according to (C.13).

F.2 Z-neutralino-neutralino interaction

The Z-neutralino-neutralino interaction Lagrangian in the rank-5 model with one singlet [with two singlets] has the form

$$\mathcal{L}_{Z\tilde\chi^0\tilde\chi^0} = \frac{1}{4}\left(g_2 W_\mu^3 - g_1 B_\mu\right)\left(\bar{\tilde{H}}_1\gamma^\mu\gamma_5\tilde{H}_1 - \bar{\tilde{H}}_2\gamma^\mu\gamma_5\tilde{H}_2\right)$$
$$+\frac{1}{4}g'Z_\mu'\left(Y_1'\bar{\tilde{H}}_1\gamma^\mu\gamma_5\tilde{H}_1 + Y_2'\bar{\tilde{H}}_2\gamma^\mu\gamma_5\tilde{H}_2\right.$$
$$\left.+Y_3'\bar{\tilde{N}}_1\gamma^\mu\gamma_5\tilde{N}_1\,[+Y_4'\bar{\tilde{N}}_2\gamma^\mu\gamma_5\tilde{N}_2]\right), \qquad \text{(F.5)}$$

whereas in the rank-6 model additional terms appear

$$+\frac{1}{4}g''Z_\mu''\left(Y_1''\bar{\tilde{H}}_1\gamma^\mu\gamma_5\tilde{H}_1 + Y_2''\bar{\tilde{H}}_2\gamma^\mu\gamma_5\tilde{H}_2\right.$$
$$\left.+Y_3''\bar{\tilde{N}}_1\gamma^\mu\gamma_5\tilde{N}_1 + Y_4''\bar{\tilde{N}}_2\gamma^\mu\gamma_5\tilde{N}_2\right). \quad \text{(F.6)}$$

In both cases it can be written as

$$\mathcal{L}_{Z\tilde\chi^0\tilde\chi^0} = \sum_{n=1}^{n_Z}\frac{1}{2}\frac{g_2}{\cos\theta_W}Z_{n,\mu}\bar{\tilde\chi}_i^0\gamma^\mu$$
$$\times\left(O_{ij}^{''nL}P_L + O_{ij}^{''nR}P_R\right)\tilde\chi_j^0 \qquad \text{(F.7)}$$

with

$$O_{ij}^{''nL} = \frac{1}{2}\left(\cos 2\beta(-N_{i3}N_{j3}^* + N_{i4}N_{j4}^*)\right.$$
$$-\sin 2\beta(N_{i3}N_{j4}^* + N_{i4}N_{j3}^*)\Big)N_{n1}^Z$$
$$-\frac{1}{2}\frac{g'}{g_1}\sin\theta_W\Big((Y_1'\cos^2\beta + Y_2'\sin^2\beta)N_{i3}N_{j3}^*$$
$$+(Y_1'\sin^2\beta + Y_2'\cos^2\beta)N_{i4}N_{j4}^*$$
$$+\frac{1}{2}(Y_1' - Y_2')\sin 2\beta(N_{i3}N_{j4}^* + N_{i4}N_{j3}^*)$$
$$+Y_3'N_{i6}N_{j6}^*\,[+Y_4'N_{i7}N_{j7}^*]\Big)N_{n2}^Z$$
$$\Big[-\frac{1}{2}\frac{g''}{g_1}\sin\theta_W\Big((Y_1''\cos^2\beta + Y_2''\sin^2\beta)N_{i3}N_{j3}^*$$
$$+(Y_1''\sin^2\beta + Y_2''\cos^2\beta)N_{i4}N_{j4}^*$$
$$+\frac{1}{2}(Y_1'' - Y_2'')\sin 2\beta(N_{i3}N_{j4}^* + N_{i4}N_{j3}^*)$$

$$+Y_3'' N_{i6} N_{j6}^* + Y_4'' N_{i7} N_{j7}^* \Big) N_{n3}^Z \Big], \tag{F.8}$$

$$O_{ij}''^{nR} = -\left(O_{ij}''^{nL}\right)^*, \tag{F.9}$$

where the diagonalization matrix N of the neutralinos is given in basis (4). In the rank-5 models it is $n_Z = 2$ with the couplings Y_i' according to (2) and the diagonalization matrix N^Z of the neutral gauge bosons according to (C.10). In the rank-6 model with $n_Z = 3$ the couplings Y_i' and Y_i'' are given in (3) and N^Z in (C.13).

F.3 Fermion-sfermion-neutralino interaction

The fermion-sfermion-neutralino interaction Lagrangian has the same form as in the MSSM

$$\mathcal{L}_{f\tilde{f}\tilde{\chi}_i^0} = g_2 f_{fi}^L \bar{f} P_R \tilde{\chi}_i^0 \tilde{f}_L + g_2 f_{fi}^R \bar{f} P_L \tilde{\chi}_i^0 \tilde{f}_R + \text{h.c.} \tag{F.10}$$

with the extended couplings in the rank-5 [rank-6] model

$$f_{fi}^L = -\sqrt{2}\Bigg(\frac{1}{\cos\theta_W}(T_{3L} - Q\sin^2\theta_W)N_{i2} + Q\sin\theta_W N_{i1}$$
$$+\frac{Y'(f_L)}{2}\frac{g'}{g_1}\tan\theta_W N_{i5}\left[+\frac{Y''(f_L)}{2}\frac{g''}{g_1}\tan\theta_W N_{i8}\right]\Bigg), \tag{F.11}$$

$$f_{fi}^R = -\sqrt{2}\sin\theta_W\Bigg(Q\tan\theta_W N_{i2}^* - Q N_{i1}^*$$
$$-\frac{Y'(f_R)}{2}\frac{g'}{g_1}\frac{1}{\cos\theta_W}N_{i5}^*\left[-\frac{Y''(f_R)}{2}\frac{g''}{g_1}\frac{1}{\cos\theta_W}N_{i8}^*\right]\Bigg). \tag{F.12}$$

f denotes the respective fermion field, $\tilde{f}_{L/R}$ the field of its scalar superpartner and N the diagonalization matrix of the neutralinos in basis (4). In the rank-5 models the $Y'(f_{L,R})$ are given in (E.4), whereas in the rank-6 model $Y'(f_{L,R})$, $Y''(f_{L,R})$ are fixed according to (E.6).

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